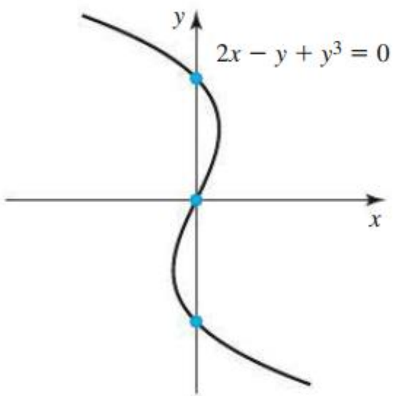
3.8 Group Activity Problems - Solutions

9. Consider the curve defined by $2x - y + y^3 = 0$ (see figure).

a. Find the coordinates of the y-intercepts of the curve.

b. Verify that $\frac{dy}{dx} = \frac{2}{1 - 3y^2}$.

c. Find the slope of the curve at each point where $x = 0$.



3.8.9

a. When $x = 0$ we have $-y + y^3 = 0$, so $y(y^2 - 1) = 0$, so y can be either 0 or ± 1. So the y-intercepts are $(0,0)$, $(0,1)$, and $(0,-1)$.

b. Differentiating both sides with respect to x gives $2 - \frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$. Thus $2 = \frac{dy}{dx} - 3y^2\frac{dy}{dx}$, so $2 = \frac{dy}{dx}(1 - 3y^2)$ and $\frac{dy}{dx} = \frac{2}{1-3y^2}$.

c. At $(0,0)$ we have $\frac{dy}{dx} = 2$, at $(0,1)$ and at $(0,-1)$ we have $\frac{dy}{dx} = \frac{2}{1-3} = -1$.

13–26. Implicit differentiation *Carry out the following steps.*

a. *Use implicit differentiation to find* $\frac{dy}{dx}$.

b. *Find the slope of the curve at the given point.*

20. $\tan xy = x + y; (0, 0)$

22. $\dfrac{x}{y^2 + 1} = 1; (10, 3)$

24. $x^{2/3} + y^{2/3} = 2; (1, 1)$

3.8.20

a. $(y + x\frac{dy}{dx})\sec^2(xy) = 1 + \frac{dy}{dx}$, so $x\frac{dy}{dx}\sec^2(xy) - \frac{dy}{dx} = 1 - y\sec^2(xy)$. Factoring out $\frac{dy}{dx}$ on the left-hand side gives
$\frac{dy}{dx}(x\sec^2(xy) - 1) = 1 - y\sec^2(xy)$, so $\frac{dy}{dx} = \frac{1 - y\sec^2(xy)}{x\sec^2(xy) - 1}$.

b. $\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{1-0}{0-1} = -1.$

3.8.22

a. We can rewrite the equation as $x = y^2 + 1$. Then $1 = 2y\frac{dy}{dx}$, so $\frac{dy}{dx} = \frac{1}{2y}$.

b. $\left.\frac{dy}{dx}\right|_{(10,3)} = \frac{1}{6}.$

3.8.24

a. $\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$, so $y^{-1/3}\frac{dy}{dx} = -\frac{1}{x^{1/3}}$, so $\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$.

b. $\left.\frac{dy}{dx}\right|_{(1,1)} = -1.$

27–40. Implicit differentiation *Use implicit differentiation to find* $\dfrac{dy}{dx}$.

32. $e^{xy} = 2y$

3.8.32 $(y + x\frac{dy}{dx})e^{xy} = 2\frac{dy}{dx}$, so $ye^{xy} + x\frac{dy}{dx}e^{xy} = 2\frac{dy}{dx}$. We can write this as $ye^{xy} = 2\frac{dy}{dx} - x\frac{dy}{dx}e^{xy}$, and factoring out the factor of $\frac{dy}{dx}$ on the right yields $ye^{xy} = \frac{dy}{dx}(2 - xe^{xy})$. Finally, we can divide to obtain $\frac{dy}{dx} = \frac{ye^{xy}}{2 - xe^{xy}}$.

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64. Vertical tangent lines

a. Determine the points where the curve $x + y^3 - y = 1$ has a vertical tangent line (see Exercise 60).

b. Does the curve have any horizontal tangent lines? Explain.

3.8.64

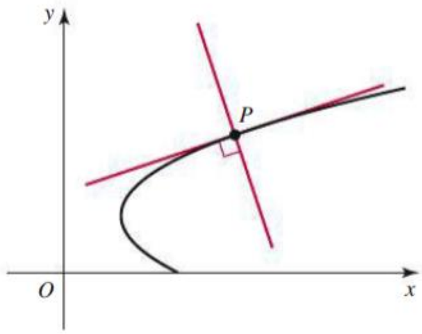
a. From exercise 60 we have that $y' = \frac{1}{1 - 3y^2}$. A vertical tangent would occur at a point whose y value would make $1 - 3y^2$ equal to zero. So we are looking for where $3y^2 = 1$ or $y = \pm\frac{1}{\sqrt{3}}$.

If $y = \frac{1}{\sqrt{3}}$, then $x + \left(\frac{1}{\sqrt{3}}\right)^3 - \frac{1}{\sqrt{3}} = 1$, so $x = 1 + \frac{2\sqrt{3}}{9}$, and there is a vertical tangent at $\left(\frac{1}{\sqrt{3}}, 1 + \frac{2\sqrt{3}}{9}\right)$.

If $y = -\frac{1}{\sqrt{3}}$, then $x + \left(-\frac{1}{\sqrt{3}}\right)^3 - -\frac{1}{\sqrt{3}} = 1$, so $x = 1 - \frac{2\sqrt{3}}{9}$, and there is a vertical tangent at $\left(-\frac{1}{\sqrt{3}}, 1 - \frac{2\sqrt{3}}{9}\right)$.

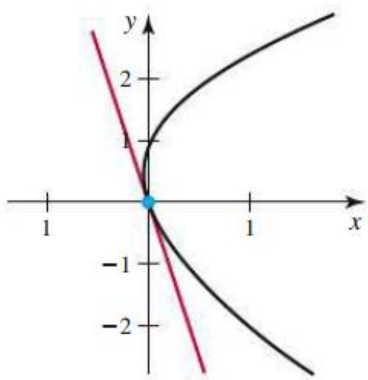
b. Because y' is never zero, there are no horizontal tangent lines.

73–78. Normal lines *A* **normal line** *at a point P on a curve passes through P and is perpendicular to the line tangent to the curve at P (see figure). Use the following equations and graphs to determine an equation of the normal line at the given point. Illustrate your work by graphing the curve with the normal line.*



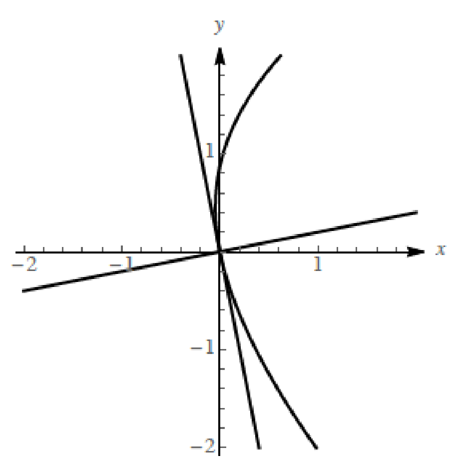
73. Exercise 45

45. $\sin y + 5x = y^2$; $(0, 0)$



3.8.73

The slope of the normal line is the negative reciprocal of the slope of the tangent line. From 45: $y' = -5$, so the slope of the normal line is $\frac{1}{5}$. At the point (0,0), we have the normal line $y = \frac{1}{5}x$.



75–86. Logarithmic differentiation *Use logarithmic differentiation to evaluate $f'(x)$.*

84. $f(x) = (1 + x^2)^{\sin x}$

85. $f(x) = \left(1 + \frac{1}{x}\right)^x$

3.9.84 Let $y = (1+x^2)^{\sin x}$. Then $\ln y = \sin x \cdot \ln(1+x^2)$, so $\frac{1}{y} \cdot y' = \cos x \cdot \ln(1+x^2) + \sin x \cdot \frac{2x}{1+x^2}$. Therefore we have $y' = (1+x^2)^{\sin x}\left(\cos x \cdot \ln(1+x^2) + \frac{2x \sin x}{1+x^2}\right)$.

3.9.85 Let $y = \left(1+\frac{1}{x}\right)^x$. Then $\ln y = x \ln\left(1+\frac{1}{x}\right)$, so $\frac{1}{y} \cdot y' = \ln\left(1+\frac{1}{x}\right) + x\left(\frac{-1/x^2}{1+1/x}\right) = \ln\left(1+\frac{1}{x}\right) - \frac{1}{x+1}$. Therefore, $y' = \left(1+\frac{1}{x}\right)^x \left(\ln\left(1+\frac{1}{x}\right) - \frac{1}{x+1}\right)$.