4.1 Group Activity Solutions

T/F Critical points may occur at the endpoints.

No! See the procedure box below as a reminder.

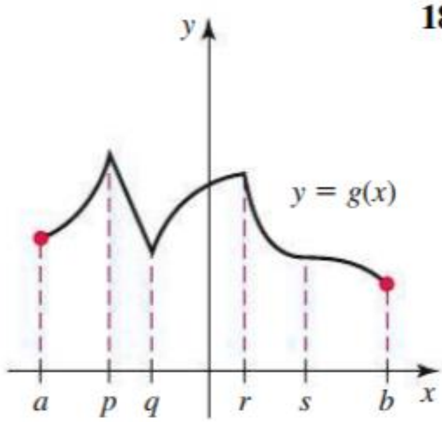
PROCEDURE Locating Absolute Extreme Values on a Closed Interval

Assume the function f is continuous on the closed interval $[a, b]$.

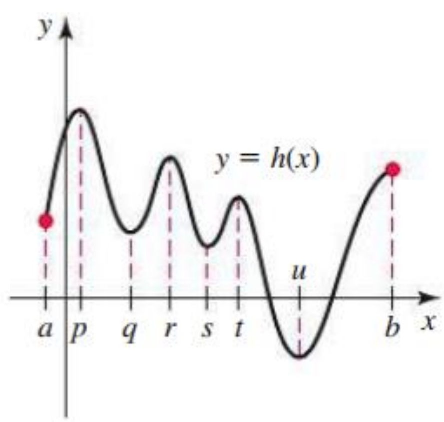
1. Locate the critical points c in (a, b), where $f'(c) = 0$ or $f'(c)$ does not exist. These points are candidates for absolute maxima and minima.
2. Evaluate f at the critical points and at the endpoints of $[a, b]$.
3. Choose the largest and smallest values of f from Step 2 for the absolute maximum and minimum values, respectively.

15–18. *Use the following graphs to identify the points on the interval $[a, b]$ at which local and absolute extreme values occur.*

17.



18.



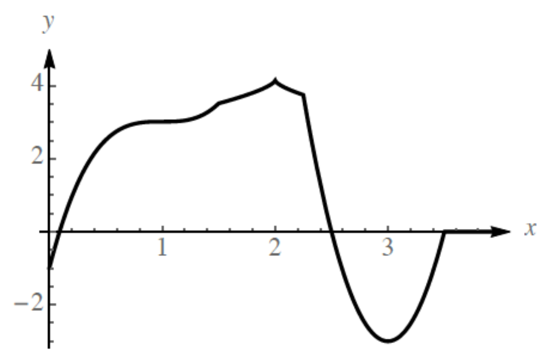
4.1.17 $y = g(x)$ has an absolute minimum at $x = b$ and an absolute maximum at $x = p$. It has local maxima at $x = p$ and $x = r$. It has a local minimum at $x = q$.

4.1.18 $y = h(x)$ has an absolute maximum at $x = p$ and an absolute minimum at $x = u$. It has local maxima at $x = p$, $x = r$ and $x = t$. It has local minima at $x = q$, $x = s$, and $x = u$.

19–22. *Sketch the graph of a continuous function f on $[0, 4]$ satisfying the given properties.*

22. $f'(x) = 0$ at $x = 1$ and 3; $f'(2)$ is undefined; f has an absolute maximum at $x = 2$; f has neither a local maximum nor a local minimum at $x = 1$; and f has an absolute minimum at $x = 3$.

4.1.22 Note the maximum at 2, and the minimum at 3. Note also the horizontal tangent lines at $x = 1$ and $x = 3$, and the sharp "corner" at $x = 2$.



23–42. Locating critical points *Find the critical points of the following functions. Assume a is a nonzero constant.*

36. $f(t) = t^2 - 2\ln(t^2 + 1)$

4.1.36 $f'(t) = 2t - \dfrac{4t}{t^2+1} = \dfrac{2t^3 - 2t}{t^2+1} = \dfrac{2t(t+1)(t-1)}{t^2+1}$. This is zero for $t = 0, \pm 1$.

40. $f(x) = \dfrac{x}{\sqrt{x-a}}$

4.1.40 $f'(x) = \dfrac{\sqrt{x-a} - \frac{x}{2\sqrt{x-a}}}{x-a} = \dfrac{\sqrt{x-a} - \frac{x}{2\sqrt{x-a}}}{x-a} \cdot \dfrac{2\sqrt{x-a}}{2\sqrt{x-a}} = \dfrac{2x-2a-x}{2(x-a)^{3/2}} = \dfrac{x-2a}{2(x-a)^{3/2}}$. This is zero when $x = 2a$, so there is a critical point at $x = 2a$ for $a > 0$.

43–68. Absolute maxima and minima *Determine the location and value of the absolute extreme values of f on the given interval, if they exist.*

60. $f(x) = 2x^6 - 15x^4 + 24x^2$ on $[-2, 2]$

4.1.60 $f'(x) = 12x^5 - 60x^3 + 48x = 12x(x^4 - 5x^2 + 4) = 12x(x^2-4)(x^2-1) = 12x(x+2)(x-2)(x+1)(x-1)$. This is zero for $x = 0$, $x = \pm 2$, and $x = \pm 1$. The critical points occur at 0 and ± 1, since the endpoints are ± 2. $f(\pm 2) = -16$, $f(\pm 1) = 11$, and $f(0) = 0$. The absolute maximum is 11 and the absolute minimum is -16.

73. Trajectory high point A stone is launched vertically upward from a cliff 192 ft above the ground at a speed of 64 ft/s. Its height above the ground t seconds after the launch is given by $s = -16t^2 + 64t + 192$, for $0 \le t \le 6$. When does the stone reach its maximum height?

4.1.73 The stone will reach its maximum height when its velocity is zero, which occurs at the only critical point for this inverted parabola. We have that $v(t) = s'(t) = -32t + 64$, which is zero when $t = 2$. The height at this time is $s(2) = 256$, the maximum height.

77. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. The function $f(x) = \sqrt{x}$ has a local maximum on the interval $[0, \infty)$.

b. If a function has an absolute maximum on a closed interval, then the function must be continuous on that interval.

c. A function f has the property that $f'(2) = 0$. Therefore, f has a local extreme value at $x = 2$.

d. Absolute extreme values of a function on a closed interval always occur at a critical point or an endpoint of the interval.

4.1.77

a. False. The derivative $f'(x) = \dfrac{1}{2\sqrt{x}}$ is never zero, and the function has no critical points.

b. False. For example, the function $f(x) = \begin{cases} \sin x & \text{if } -5 \le x \le 0, \\ -8 & \text{if } 0 < x \le 5 \end{cases}$ is not continuous on $[-5, 5]$, but has an absolute maximum of 1.

c. False. For example, the function $f(x) = (x-2)^3$ satisfies $f'(2) = 0$, but it has neither a maximum nor a minimum at $x = 2$.

d. True. This follows from the theorems in this section.