4.6 Group Activity Problems - Solutions

6. Suppose f is differentiable on $(-\infty, \infty)$ and the equation of the line tangent to the graph of f at $x = 2$ is $y = 5x - 3$. Use the linear approximation to f at $x = 2$ to approximate $f(2.01)$.

4.6.6 $L(x) = 5x - 3$, so $f(2.01) \approx 5(2.01) - 3 = 7.05$.

11. Suppose f is differentiable on $(-\infty, \infty)$ and $f(5.01) - f(5) = 0.25$. Use linear approximation to estimate the value of $f'(5)$.

4.6.11 $\Delta y \approx f'(a)\Delta x$, so $f'(a) \approx \dfrac{\Delta y}{\Delta x}$. Therefore $f'(5) \approx \dfrac{f(5.01) - f(5)}{5.01 - 5} = \dfrac{0.25}{0.01} = 25$.

19–24. Linear approximation *Find the linear approximation to the following functions at the given point a.*

22. $h(w) = \sqrt{5w - 1}; a = 1$

4.6.22 $h'(w) = \dfrac{5}{2\sqrt{5w-1}}$, so $h'(1) = \dfrac{5}{4}$.

$$L(w) = h(1) + h'(1)(w-1) = 2 + \frac{5}{4}(w-1) = \frac{5}{4}w + \frac{3}{4}.$$

T 25–36. Linear approximation

a. *Write the equation of the line that represents the linear approximation to the following functions at the given point a.*

b. *Use the linear approximation to estimate the given quantity.*

c. *Compute the percent error in your approximation,* $100|\text{approximation} - \text{exact}|/|\text{exact}|$, *where the exact value is given by a calculator.*

35. $f(x) = e^{-x}; a = 0; e^{-0.03}$

4.6.35

a. Note that $f(a) = f(0) = 1$ and $f'(a) = -e^{-a} = -1$, so the linear approximation has equation

$$y = L(x) = f(a) + f'(a)(x - a) = 1 - x.$$

b. The linear approximation to $e^{-0.03}$ is $e^{-0.03} \approx L(0.03) = 0.97$.

c. The percentage error is $100 \cdot \dfrac{|0.97 - e^{-0.03}|}{e^{-0.03}} \approx 0.046\%$.

MARGINAL ANALYSIS

3.6	Calculate and interpret average and marginal cost	31, 32, 108 on p. 240

29–32. Average and marginal cost *Consider the following cost functions.*

a. *Find the average cost and marginal cost functions.*

b. *Determine the average cost and the marginal cost when $x = a$.*

c. *Interpret the values obtained in part (b).*

31. $C(x) = -0.01x^2 + 40x + 100, 0 \le x \le 1500, a = 1000$

32. $C(x) = -0.04x^2 + 100x + 800, 0 \le x \le 1000, a = 500$

3.6.31

a. The average cost function is given by $\overline{C}(x) = \frac{C(x)}{x} = \frac{100}{x} + 40 - 0.01x$. The marginal cost function is given by $M(x) = C'(x) = 40 - 0.02x$.

b. At $a = 1000$ we have $\overline{C}(1000) = \frac{100}{1000} + 40 - (.01)(1000) = 30.1$, and $M(1000) = 20$.

c. The average cost per item when producing 1000 items is \$30.10. The cost of producing the next item is \$20.00.

3.6.32

a. The average cost function is given by $\overline{C}(x) = \frac{C(x)}{x} = \frac{800}{x} + 100 - 0.04x$. The marginal cost function is given by $M(x) = C'(x) = 100 - 0.08x$.

b. At $a = 500$ we have $\overline{C}(500) = \frac{800}{500} + 100 - (.04)(500) = 81.6$, and $M(500) = 60$.

c. The average cost per item when producing 500 items is \$81.60. The cost of producing the next item is \$60.00.

108. Marginal and average cost Assume $C(x) = -0.0001x^3 + 0.05x^2 + 60x + 800$ is the cost of making x fly rods.

a. Determine the average and marginal costs for $x = 400$ fly rods.

b. Interpret the meaning of your results in part (a).

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a. The marginal cost is given by $C'(x) = -0.0003x^2 + 0.1x + 60$, so $C'(400) = \$52$. The average cost of producing 400 fly rods is $\dfrac{C(400)}{400} = \$66$.

b. The average cost of producing 400 fly rods is \$66 per fly rod. The cost of producing the 401st fly rod is approximately \$52.