

## 4.6 linear approximation, marginal analysis Group Activity Problems - Solutions

This approximation improves as  $x$  approaches  $a$ .

### **DEFINITION** Linear Approximation to $f$ at $a$

Suppose  $f$  is differentiable on an interval  $I$  containing the point  $a$ . The **linear approximation** to  $f$  at  $a$  is the linear function

$$L(x) = f(a) + f'(a)(x - a), \quad \text{for } x \text{ in } I.$$

6. Suppose  $f$  is differentiable on  $(-\infty, \infty)$  and the equation of the line tangent to the graph of  $f$  at  $x = 2$  is  $y = 5x - 3$ . Use the linear approximation to  $f$  at  $x = 2$  to approximate  $f(2.01)$ .



11. Suppose  $f$  is differentiable on  $(-\infty, \infty)$  and  $f(5.01) - f(5) = 0.25$ . Use linear approximation to estimate the value of  $f'(5)$ .

**19–24. Linear approximation** Find the linear approximation to the following functions at the given point  $a$ .

22.  $h(w) = \sqrt{5w - 1}; a = 1$

**25–36. Linear approximation**

- Write the equation of the line that represents the linear approximation to the following functions at the given point  $a$ .
- Use the linear approximation to estimate the given quantity.
- Compute the percent error in your approximation,  $100 \left| \frac{\text{approximation} - \text{exact}}{\text{exact}} \right|$ , where the exact value is given by a calculator.

35.  $f(x) = e^{-x}; a = 0; e^{-0.03}$

## MARGINAL ANALYSIS

3.6 Calculate and interpret average and marginal cost

31, 32, 108 on p. 240

**29–32. Average and marginal cost** Consider the following cost functions.

- Find the average cost and marginal cost functions.
- Determine the average cost and the marginal cost when  $x = a$ .
- Interpret the values obtained in part (b).

**31.**  $C(x) = -0.01x^2 + 40x + 100, 0 \leq x \leq 1500, a = 1000$

**32.**  $C(x) = -0.04x^2 + 100x + 800, 0 \leq x \leq 1000, a = 500$

**108. Marginal and average cost** Assume  $C(x) = -0.0001x^3 + 0.05x^2 + 60x + 800$  is the cost of making  $x$  fly rods.

- Determine the average and marginal costs for  $x = 400$  fly rods.
- Interpret the meaning of your results in part (a).