5.1-5.2 Group Activity Problems Solutions

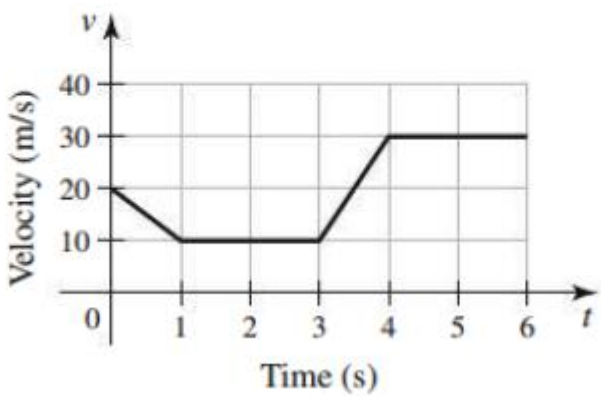
70. Displacement from a velocity graph Consider the velocity f
tion for an object moving along a line (see figure).

a. Describe the motion of the object over the interval $[0, 6]$.

b. Use geometry to find the displacement of the object between $t = 0$ and $t = 3$.

c. Use geometry to find the displacement of the object between $t = 3$ and $t = 5$.

d. Assuming the velocity remains 30 m/s, for $t \geq 4$, find the function that gives the displacement between $t = 0$ and any time $t \geq 4$.



5.1.70

a. The object's velocity decreases during the first second, then remains constant between time $t = 1$ and $t = 3$, and then steadily increases until $t = 4$, and then stays constant after that.

b. The displacement is given by the area under the curve, which between $t = 0$ and $t = 3$ is 35, so the displacement is 35 meters.

c. Between $t = 3$ and $t = 5$ the area under the curve is 50, so the displacement is 50 meters.

d. Between $t = 0$ and $t = 4$ the displacement is 55, and between 4 and t for $t > 4$, the displacement is $30(t - 4)$. So the displacement between 0 and t for $t > 4$ is $55 + 30(t - 4)$.

10. Suppose $\int_1^3 f(x)\,dx = 10$ and $\int_1^3 g(x)\,dx = -20$. Evaluate $\int_1^3 (2f(x) - 4g(x))\,dx$ and $\int_3^1 (2f(x) - 4g(x))\,dx$.

5.2.10 $\displaystyle\int_1^3 (2f(x) - 4g(x))\,dx = 2\int_1^3 f(x)\,dx - 4\int_1^3 g(x)\,dx = 2(10) - 4(-20) = 100.$

$\displaystyle\int_3^1 (2f(x) - 4g(x))\,dx = -\int_1^3 (2f(x) - 4g(x))\,dx = -100.$

15. Use geometry to find a formula for $\int_0^a x\,dx$, in terms of a constant $a > 0$.

16. If f is continuous on $[a, b]$ and $\int_a^b |f(x)|\,dx = 0$, what can you conclude about f?

5.2.15 This integral represents the area under $y = x$ between $x = 0$ and $x = a$, which is a right triangle. The length of the base of the triangle is a and the height is a, so the area is $\frac{1}{2} \cdot a^2$, so $\displaystyle\int_0^a x\,dx = \frac{a^2}{2}$.

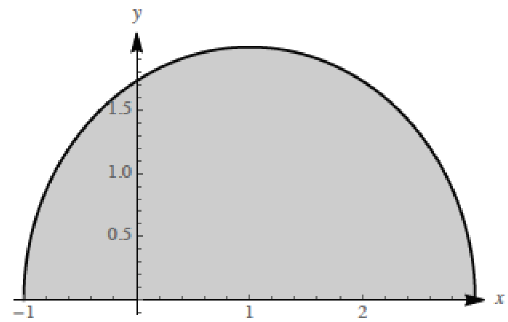
5.2.16 Because the function $|f|$ never goes below the x axis, the definite integral of $|f|$ does represent the area between $|f|$ and the x-axis. If this area is zero, then f must strictly lie on the x axis, so f must be the constant function with value 0.

39–46. Definite integrals *Use geometry (not Riemann sums) to evaluate the following definite integrals. Sketch a graph of the integrand, show the region in question, and interpret your result.*

44. $\int_{-1}^{3} \sqrt{4 - (x - 1)^2}\, dx$

5.2.44

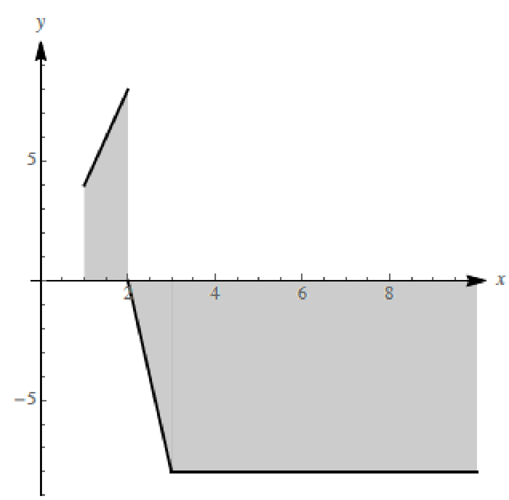
The region consists of a semicircle situated above the axis, of radius 2. The area is thus $\frac{4\pi}{2} = 2\pi$.



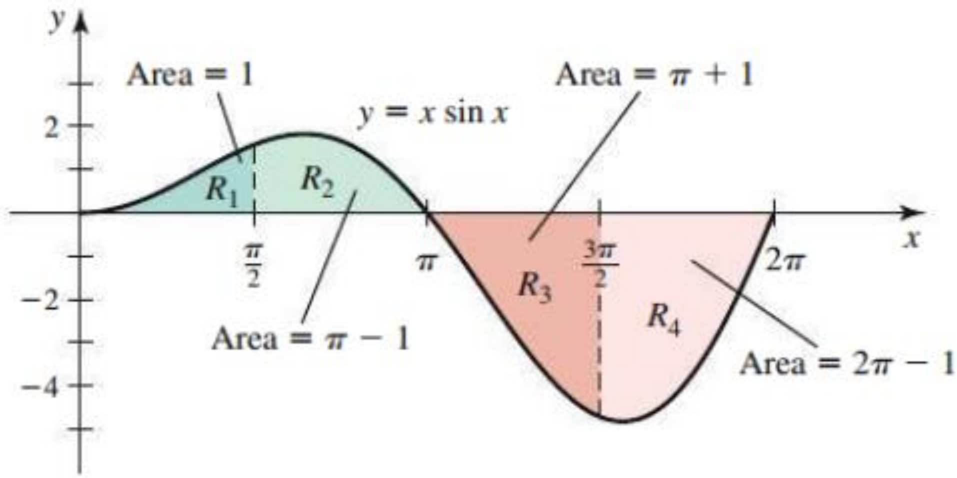
46. $\int_{1}^{10} g(x)\, dx$, where $g(x) = \begin{cases} 4x & \text{if } 0 \le x \le 2 \\ -8x + 16 & \text{if } 2 < x \le 3 \\ -8 & \text{if } x > 3 \end{cases}$

5.2.46

The region consists of a trapezoid of area 6 above the axis, a triangle of area 4 below the axis, and a rectangle of area 56 below the axis. So the net area is $6 - 4 - 56 = -54$.



47–50. *The accompanying figure shows four regions bounded by the graph of* $y = x \sin x$: R_1, R_2, R_3, *and* R_4, *whose areas are* 1, $\pi - 1$, $\pi + 1$, *and* $2\pi - 1$, *respectively. (We verify these results later in the text.) Use this information to evaluate the following integrals.*



47. $\int_0^{\pi} x \sin x \, dx$

48. $\int_0^{3\pi/2} x \sin x \, dx$

49. $\int_0^{2\pi} x \sin x \, dx$

50. $\int_{\pi/2}^{2\pi} x \sin x \, dx$

5.2.47 $\int_0^{\pi} x \sin x \, dx = A(R_1) + A(R_2) = 1 + \pi - 1 = \pi.$

5.2.48 $\int_0^{3\pi/2} x \sin x \, dx = A(R_1) + A(R_2) - A(R_3) = 1 + \pi - 1 - \pi - 1 = -1.$

5.2.49 $\int_0^{2\pi} x \sin x \, dx = A(R_1) + A(R_2) - A(R_3) - A(R_4) = 1 + \pi - 1 - \pi - 1 - 2\pi + 1 = -2\pi.$

5.2.50 $\int_{\pi/2}^{2\pi} x \sin x \, dx = A(R_2) - A(R_3) - A(R_4) = \pi - 1 - \pi - 1 - 2\pi + 1 = -2\pi - 1.$

67. Use geometry and properties of integrals to evaluate $\int_0^1 (2x + \sqrt{1 - x^2} + 1)\, dx$.

5.2.67 $\int_0^1 (2x + \sqrt{1-x^2} + 1)\, dx = \int_0^1 2x\, dx + \int_0^1 \sqrt{1-x^2}\, dx + \int_0^1 1\, dx$. The first integral in this sum represents the area of a triangle with base 1 and height 2 (which has value 1), the second represents the area of a quarter of a circle of radius 1 (which has value $\frac{\pi}{4}$, and the third represents the area of a 1×1 square (which has value 1). So the integral's value is $1 + \frac{\pi}{4} + 1 = 2 + \frac{\pi}{4}$.