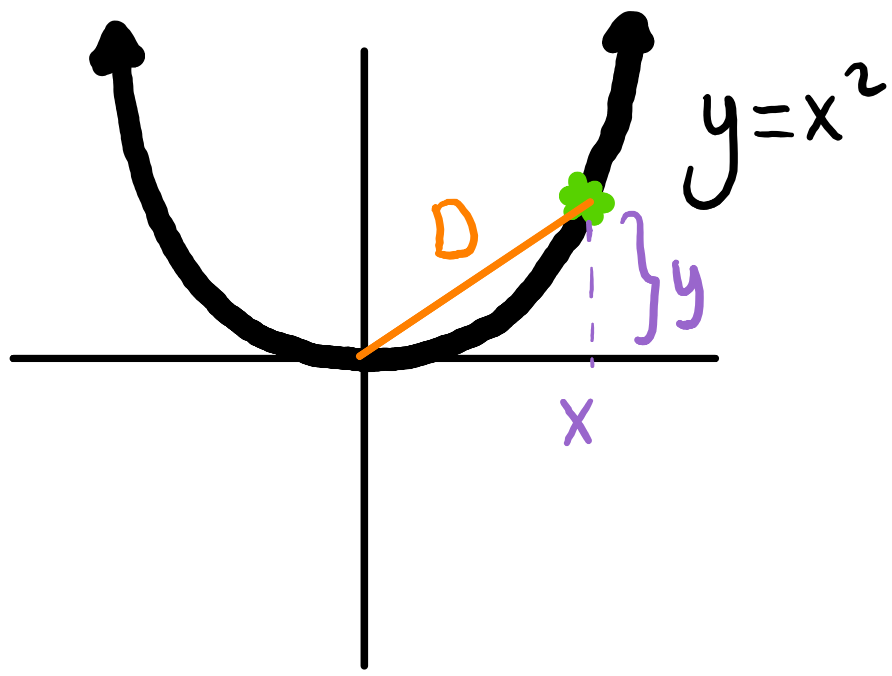
3.11 Related Rates

A bug is moving along the right side of the parabola $y = x^2$ at a rate such that its distance from the origin is increasing at 8 cm / min.

a. At what rate is the x-coordinate of the bug increasing when the bug is at the point (4, 16)?

b. Use the equation $y = x^2$ to find an equation relating $\frac{dy}{dt}$ to $\frac{dx}{dt}$.

c. At what rate is the y-coordinate of the bug increasing when the bug is at the point (4, 16)?



Given: $\frac{dD}{dt} = +8 \frac{cm}{min}$; $D^2 = x^2 + y^2$

Asked: when $(x,y) = (4,16)$

$\frac{dx}{dt} = ?$ $\quad \frac{dy}{dt} = ?$

relate

$$\left.\begin{array}{l} D^2 = x^2 + y^2 \\ \text{since } y = x^2 \end{array}\right\}$$

$$D^2 = x^2 + (x^2)^2 = x^2 + x^4 \Rightarrow D = \sqrt{x^2 + x^4}$$

$$\underbrace{2D \cdot \frac{dD}{dt} = 2x \cdot \frac{dx}{dt} + 4x^3 \cdot \frac{dx}{dt}}$$

Der. wrt time

when $x=4, y=16$ $\quad D^2 = 4^2 + 16^2 = 272 \Rightarrow D = \sqrt{272}$ $\quad$ subs #

$$\boxed{\text{Simplified as: } \frac{dD}{dt} = \frac{\left(x + 2x^3\right)}{\sqrt{x^2+x^4}} \cdot \frac{dx}{dt}}$$

$$2 \cdot \sqrt{272} \cdot 8 = \overset{8}{2 \cdot 4} \cdot \frac{dx}{dt} + \overset{256}{4 \cdot 4^3} \cdot \frac{dx}{dt}$$

$$\overset{2}{\cancel{16}}\sqrt{272} = \frac{\cancel{264}}{33} \frac{dx}{dt} \Rightarrow \boxed{\frac{dx}{dt} = \frac{2\sqrt{272}}{33}}$$

b) $y = x^2 \Rightarrow \dfrac{dy}{dt} = 2 \cdot x \cdot \dfrac{dx}{dt}$

c) From part a)

$\dfrac{dx}{dt} = \dfrac{2\sqrt{272}}{33}$

$x = 4$ cm (given)

$$\frac{dy}{dt} = 2 \cdot 4 \cdot \frac{2\sqrt{272}}{33} = \frac{16\sqrt{272}}{33} \ \frac{\text{cm}}{\text{min}}.$$