Exp) Given $f(x)=|x|$, calculate $f'(-4)$, $f'(0)$.

Recall: $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

$$f'(-4)=\lim_{h\to 0}\frac{f(-4+h)-f(-4)}{h}$$

$f(-4)=|-4|=4$

$f(-4+h)=|-4+h|$

recall: $h\to 0$

therefore, $-4+h<0$ and

$|-4+h|=-(-4+h)$

$$=\lim_{h\to 0}\frac{-(-4+h)-4}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{4}-\cancel{h}^{-1}-\cancel{4}}{\cancel{h}}=-1$$

remember $f(x)=|x|\begin{cases}x, & x\geq 0\\ -x, & x<0\end{cases}$

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\underbrace{\lim_{h\to 0}\frac{|h|}{h}}_{\text{2 sided limit, check both 1 sided limits!}}=?$$

$$\lim_{h\to 0^-}\frac{|h|}{h}=\lim_{h\to 0^-}\left(\frac{-h}{h}\right)=-1$$

as $h\to 0^-$, $h<0$, $|h|=-h$

$$\lim_{h\to 0^+}\frac{|h|}{h}=\lim_{h\to 0^+}\left(\frac{h}{h}\right)=1$$

as $h\to 0^+$, $h>0$, $|h|=h$

$$\lim_{h\to 0^-}\frac{|h|}{h}\neq\lim_{h\to 0^+}\frac{|h|}{h}\text{; therefore, }\lim_{h\to 0}\frac{|h|}{h}=f'(0)\text{ DNE.}$$

$$-1\neq 1$$

recall: $f(x)=|x|$ has a sharp curve at $x=0$!