Official List of Problem - 3.9

Q85) Use log. diff. to evaluate $f'(x)$

$$f(x)=\left(1+\frac{1}{x}\right)^x$$

$$\ln(f(x))=\ln\left(1+\frac{1}{x}\right)^x$$

$$\ln(f(x))=x\cdot\ln\left(1+\frac{1}{x}\right)$$

$$\frac{f'(x)}{f(x)}=1\cdot\ln\left(1+\frac{1}{x}\right)+\cancel{x}\cdot\underbrace{\frac{\left(1+\frac{1}{x}\right)'}{1+\frac{1}{x}}}_{}$$

$$\frac{(1+x^{-1})'}{\frac{x+1}{x}}=\frac{-x^{-2}}{\frac{x+1}{x}}=\frac{-1/x^{\cancel{2}}}{x+1/\cancel{x}}=\frac{-1}{\cancel{x}(x+1)}$$

$$f(x)\cdot\frac{f'(x)}{f(x)}=\left[\ln\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right]\cdot f(x)$$

$$\boxed{f'(x)=\left[\ln\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right]\cdot\left(1+\frac{1}{x}\right)^x}$$

Official List of Problems – 3.8

Q66) Find the eq. of the vertical and horizontal tangent lines for $x^2+4y^2+2xy=12$

Implicit diff: $2x+\underbrace{8y\cdot y'+2xy'}+2y=0$

$$2x+y'(8y+2x)+2y=0$$

$$y'=\frac{-2y-2x}{8y+2x}$$

Horizontal tangent line $\Rightarrow y'=0$

$y'=0 \Rightarrow -2y-2x=0 \Rightarrow x=-y$

Use the original eq: $x^2+4y^2+2xy=12$

$$(-y)^2+4y^2+2(-y)\cdot y=12$$

$$y^2+4y^2-2y^2=3y^2=12 \Rightarrow y=\pm 2$$

Vertical tangent line $\Rightarrow y'$ undef. (denominator of $y'=0$)

$8y+2x=0 \Rightarrow x=-4y$

Use the original eq: $x^2+4y^2+2xy=12$

$$(-4y)^2+4y^2+2(-4y)\cdot y=12$$

$$16y^2+4y^2-8y^2=12 \Rightarrow 12y^2=12 \Rightarrow y=\pm 1$$

when $y=1 \Rightarrow x=-4\cdot 1=-4$; $y=-1 \Rightarrow x=-4\cdot -1=4$; $x=\pm 4$