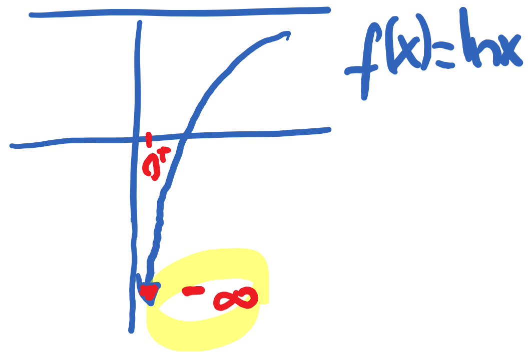
Recall:



THEOREM 4.13 L'Hôpital's Rule (∞/∞)

Suppose f and g are differentiable on an open interval I containing a, with $g'(x) \neq 0$ on I when $x \neq a$. If $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limit on the right exists (or is $\pm\infty$). The rule also applies for $x \to \pm\infty$, $x \to a^+$, or $x \to a^-$.

L.R $\neq$ Quotient Rule

EXAMPLE 3 **L'Hôpital's Rule for ∞/∞** Evaluate the following limits.

b. $$\lim_{x\to 0^+} \frac{\ln x}{\csc x} \overset{\text{"DSP"}}{=} \frac{\ln(0^+)}{\csc(0^+)} = \frac{-\infty}{\infty} \quad \text{L.R} \checkmark$$

$$\overset{H}{=} \lim_{x\to 0^+} \left(\frac{(\ln x)'}{(\csc x)'} \right) = \lim_{x\to 0^+} \left(\frac{\frac{1}{x}}{-\csc x \cdot \cot x} \right)$$

$$\csc x = \frac{1}{\sin x}$$

re-write

$$= \lim_{x\to 0^+} \left(\frac{\frac{1}{x}}{-\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}} \right) = \lim_{x\to 0^+} \left(\frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}} \right)$$

$$\csc(0^+) = \frac{1}{\sin(0^+)} = \frac{1}{0^+} = +\infty$$

"non-zero $\frac{\#}{0}$"

$$= \lim_{x\to 0^+} \left(\frac{1}{x} \cdot \frac{\sin^2 x}{-\cos x} \right) = \lim_{x\to 0^+} \left(\frac{\sin^2 x}{-x\cdot\cos x} \right) \overset{\text{"DSP"}}{=} \text{"}\frac{0}{0}\text{"}$$

$\hookrightarrow$ indet. form

use L.R.

$$\overset{H}{=} \lim_{x\to 0^+} \left(\frac{2\cdot \sin x \cdot \cos x}{-1\cdot\cos x - x\cdot(-\sin x)} \right) \overset{\text{"DSP"}}{=} \frac{0}{-1+0\cdot 0} = \frac{0}{-1} \boxed{=0}$$