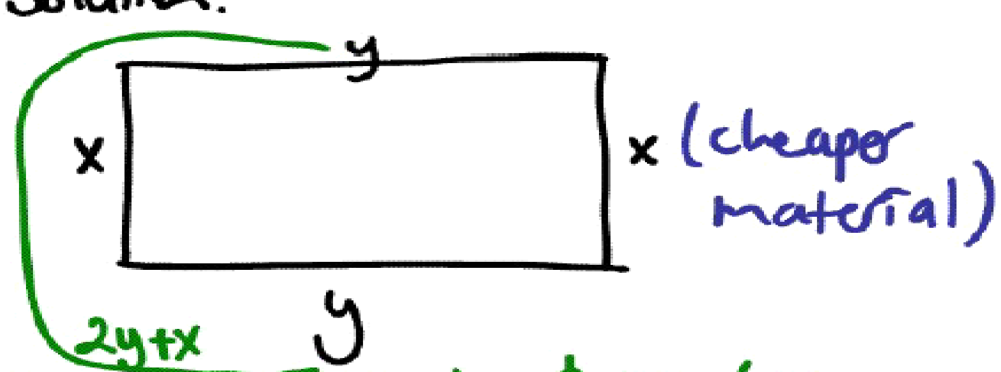
Optimization

Objective Function

Constraint

A fence must be built around a rectangular area of 1600 sq. ft. The fence along three sides is to be made of material that costs \$120/ft. The material for the 4th side costs \$40/ft. Find the dimensions (round to the nearest foot) of the rectangle that would be the least expensive to build.

Solution:



\$40/ft.

expensive material \$120/ft.

Objective F $C(x,y) = (2y+x)\cdot 120 + 40x$

Constraint $x \cdot y = 1600$

Use constraint to get: $y = \frac{1600}{x}$

re-write obj. F. $C(x) = \left(2\cdot\frac{1600}{x} + x\right)\cdot 120 + 40x$

Goal: Find x that gives the min $C(x)$

Domain of x: $x \geq 0$ $[0, \infty)$ (can't be neg.)

$C'(x) = 0$ or DNE

$C'(x) = 160 + 3200\cdot 120\cdot(-1)\cdot x^{-2} \stackrel{?}{=} 0$ or DNE

$$C'(x) = 0 \Rightarrow \quad x \approx 49 \text{ ft.}$$

$$y = \frac{1600}{x} \approx 33 \text{ ft.}$$

Is $x = 49$ ft. really the global min?

$$C''(x) = 3200 \cdot 120 \cdot (-1) \cdot (-2) \cdot x^{-3} > 0 \quad \cup$$

YES!

$x \approx 49$ ft.

The dimensions that would give the cheapest cost to build are: x = 49 ft and y= 33ft (rounded to the nearest foot).