Shipping Crates 1

21. Shipping crates A square-based, box-shaped shipping crate is designed to have a volume of 16 ft^3. The material used to make the base costs twice as much (per square foot) as the material in the sides, and the material used to make the top costs half as much (per square foot) as the material in the sides. What are the dimensions of the crate that minimize the cost of materials?

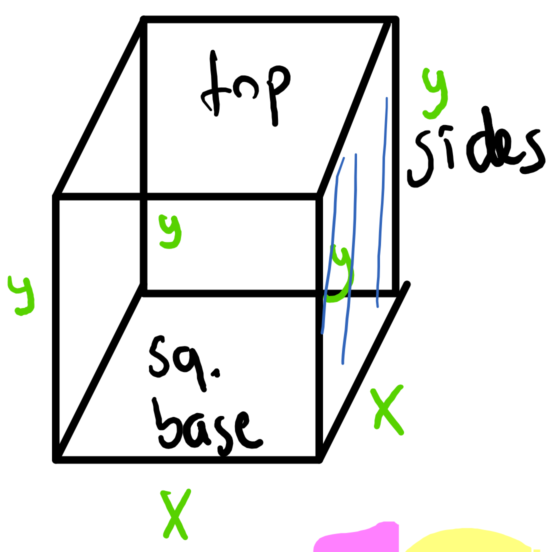
Obj. F. → 2 var. func.
Constraint eq.
Abs. min/max
Local → abs.

Sq. based box $V \to 16\ ft^3$.

diff. materials for base, sides, top

Obj? MIN. Cost of the materials

4 sides



$$V = x \cdot x \cdot y = x^2 \cdot y = 16\ ft^3$$

Constraint eq.

$C \to$ cost (per sq. ft.) of the material to make the sides

Sq. ft.

Base mat. costs 2 x mat. for the sides

Top mat. costs 1/2 x mat. for the sides

Area of a base $\to x^2$ $\quad 2c \cdot x^2$

4 x Area of the sides $\to x \cdot y \to c \cdot x \cdot y$

Area of the top $\to x^2 \to \frac{c}{2} \cdot x^2$

+ Total

Obj. F. $\text{Total Cost} = \underbrace{2c \cdot x^2}_{\text{Cost for the base}} + \underbrace{4 \cdot c \cdot xy}_{\text{Cost for all sides}} + \underbrace{\frac{c}{2} \cdot x^2}_{\text{Cost for the top}}$ MIN.

Shipping Crates 2

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Obj. F. $\text{Total Cost} = \underbrace{2c \cdot x^2}_{\text{cost for the base}} + \underbrace{4 \cdot c \cdot xy}_{\text{cost for all sides}} + \underbrace{\frac{c}{2} \cdot x^2}_{\text{cost for the top}}$ MIN.

Constraint eq. $x^2 \cdot y = 16 \rightarrow y = \frac{16}{x^2}$

$c \rightarrow$ constant

Re-write the cost (obj.) f.

$$C(x,y) = \underbrace{2cx^2 + \frac{c}{2}x^2 + 4cxy}$$

$\hookrightarrow C(x)$

$$C(x) = 2c \cdot x^2 + \frac{c}{2} \cdot x^2 + 4c \cdot x \cdot \frac{16}{x^2}$$

$$= \frac{5}{2}cx^2 + \frac{64c}{x} = c\left(\frac{5x^2}{2} + \frac{64}{x}\right)$$

Shipping Crates 3

$C'(x) = 0$ or DNE to find crit. points

$$C'(x) = c\left(\frac{5}{\cancel{2}}\cdot\cancel{2}x + 64\cdot(-1)\cdot x^{-2}\right)$$

$$= c\left(5x - 64x^{-2}\right) = 0 \text{ or DNE}$$

$$C'(x) = \underbrace{c\left(5x - \frac{64}{x^2}\right)} = c\left(\frac{5x^3-64}{x^2}\right) = 0 \text{ or DNE}$$

$5x^3 - 64 = 0$

$5x^3 = 64 \Rightarrow x^3 = \frac{64}{5} \Rightarrow \underbrace{x = \sqrt[3]{\frac{64}{5}} = \frac{4}{\sqrt[3]{5}}}_{\text{critical P.}}$; $\cancel{x = 0}$

Interval $(0, \infty)$

not in the interval!

$x = \frac{4}{\sqrt[3]{5}} \rightarrow$ crit. P.

sign chart for $C'(x)$

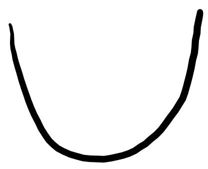
$$C'(x) = c\left(\frac{5x^3-64}{x^2}\right)$$

	0	1	$\frac{4}{\sqrt[3]{5}}$	100	∞
$C'(x)$		$-$		$+$	
$C(x)$			local min		

local min at $x = \frac{4}{\sqrt[3]{5}}$

Shipping Crates 4

$$C''(x) = c\left(5 - 64\cdot(-2)\cdot x^{-3}\right) = c\left(\overbrace{5 + \frac{128}{x^3}}^{\text{pos.}}\right) > 0$$

$C(x)$  concave up

since concave up; local min is also the global min

$x^2 \cdot y = 16 \text{ ft}^3 \Rightarrow$ use the constraint to find y

$x = \dfrac{4}{\sqrt[3]{5}}$

$$x^2 y = 16$$

$$\left(\frac{4}{5^{1/3}}\right)^2 \cdot y = 16 \Rightarrow \frac{\cancel{16}}{5^{2/3}} \cdot y = \cancel{16}$$

$$y = 5^{2/3} \text{ ft.}$$

The dimensions of the crate that MINIMIZE the cost of the materials are $\dfrac{4}{\sqrt[3]{5}}$ by $5^{2/3}$ ft.