Exp) A particle moves on a coordinate line w/ acceleration $\frac{d^2s}{dt^2} = 45\sqrt{t} - \frac{24}{\sqrt{t}}$, subject to the conditions that $\overset{v(t)}{\frac{ds}{dt}} = 8$ and $s = 17$ when $t = 1$. Find the velocity and position in terms of t.

Given

$a(t) = 45 \cdot t^{1/2} - 24 \cdot t^{-1/2}$

$v(t) = 8$, $s(t) = 17$ @ $t = 1$

$v(1) = 8$, $s(1) = 17$

Asked

$v(t) = ?$, $s(t) = ?$

$$v(t) = \int a(t) \cdot dt = \int \left(45 \cdot t^{1/2} - 24 \cdot t^{-1/2}\right) dt$$

$$= \overset{15}{\cancel{45}} \cdot \frac{t^{3/2}}{\overset{1}{\cancel{3}}/2} - \overset{48}{24} \cdot \frac{t^{1/2}}{1/2} + C_1$$

$$= 30t^{3/2} - 48t^{1/2} + C_1$$

Given: $v(1) = 8 \Rightarrow v(1) = 30 \cdot 1^{3/2} - 48 \cdot 1^{1/2} + C_1 = 8$

$$= -18 + C_1 = 8 \Rightarrow \boxed{C_1 = 26}$$

$$v(t) = 30t^{3/2} - 48t^{1/2} + 26$$

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Given
$a(t) = 45 \cdot t^{1/2} - 24 \cdot t^{-1/2}$
$v(t) = 8,\ s(t) = 17$ @ $t = 1$
$(1) = 8,\ s(1) = 17$

$s(t) = ?$

$$s(t) = \int v(t) \cdot dt \Rightarrow s(t) = \int (30t^{3/2} - 48t^{1/2} + 26) \cdot dt$$

$$s(t) = \cancel{30}^{6} \cdot \frac{t^{5/2}}{\cancel{5}/2} - \cancel{48}^{16} \cdot \frac{t^{3/2}}{\cancel{3}/2} + 26t + C_2$$

$$= 12t^{5/2} - 32t^{3/2} + 26t + C_2$$

$$s(1) = 17 \Rightarrow s(1) = 12 \underbrace{-32}_{-20} + 26 + C_2 = 17$$

$$6 + C_2 = 17 \Rightarrow \boxed{C_2 = 11}$$

$$s(t) = 12t^{5/2} - 32t^{3/2} + 26t + 11$$