

$$y = 2(4x)^x \text{ on } [0.05, 1]$$

Determine the absolute min/max.

Find critical points by $f'(x) = 0$ or DNE on $(0.05, 1)$

$$\frac{y}{2} = (4x)^x \Rightarrow \ln\left(\frac{y}{2}\right) = \ln(4x)^x$$

$$\ln\left(\frac{y}{2}\right) = x \cdot \ln(4x)$$

Find the derivative of both sides.

$$\left[\ln\left(\frac{y}{2}\right)\right]' = \left[x \cdot \ln(4x)\right]'$$

$$\frac{y'/2}{y/2} = 1 \cdot \ln(4x) + x \cdot \frac{4}{4x} = \ln(4x) + 1$$

$$\frac{y'}{y} = \ln(4x) + 1 \Rightarrow y' = y(\ln(4x) + 1)$$

$$y' = f'(x) = 2 \cdot (4x)^x \cdot (\ln(4x) + 1)$$

$$f'(x) = 2 \cdot (4x)^x \cdot (\ln(4x) + 1) = 0 \text{ or DNE}$$

$$\ln(4x) + 1 = 0 \Rightarrow \ln(4x) = -1 \Rightarrow 4x = e^{-1} \Rightarrow x = \frac{e^{-1}}{4}$$

$$x = \frac{1}{4e} \approx 0.0919 \text{ is on } (0.05, 1)$$

Find $f(0.05)$, $f(1)$, $f(\frac{1}{4e})$ to determine abs. min/max.

x	$y = 2(4x)^x$
$\frac{1}{4e}$	$2\left(4 \cdot \frac{1}{4e}\right)^{\frac{1}{4e}} = 2 \cdot \left(\frac{1}{e}\right)^{\frac{1}{4e}} \approx 1.82426$ ABS MIN
0.05	$2(4 \cdot 0.05)^{0.05} = 2(0.2)^{0.05} \approx 1.84536$
1	$2(4)^1 = 8$ ABS MAX

Recall: $y = 2 \left(\frac{1}{e}\right)^{\frac{1}{4e}} = 2 \left[\left(\frac{1}{e}\right)^{\frac{1}{4}}\right]^{\frac{1}{e}} = 2 \cdot \left(4\sqrt{\frac{1}{e}}\right)^{\frac{1}{e}}$